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Cubesat Drag Calculations

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This document describes the simple approach to estimate the orbital lifetime of an object in a circular orbit at a given altitude. It combines the method from [1] with an altitude calibration based on Figure B-1 from [2].

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Atmospheric Model (lifted from [1])

The following simplifications and assumptions have been made. The atmospheric density ρ is a simple exponential with a varying scale height H. For a fixed exospheric temperature T, H is made to vary with altitude h through the use of an effective atmospheric molecular mass m. This m includes both the actual variation in molecular mass with height and a compensation term for the variation in temperature over the altitude range under consideration. It should be noted that the original upper altitude range from [1] is limited to 500 km, but we found good agreement between orbital lifetime estimates extended to 700 km altitudes and values plotted in the ISO document on orbital lifetimes [2].

The two variables for orbital environmental changes ("space weather") that are added to this are the Solar radio flux index *F10.7* and the geomagnetic index *Ap*. The former is a proxy for the Solar X-ray output that heats the base of the thermosphere (at 120 km) which gives rise to a direct heating effect that propagates itself upward from this level. The typical value of *F10.7* varies between lows of about 65 Solar Flux Units (SFU, 10^{-22} W/m^2/Hz = 10000 Jy) to highs over 300 SFU. The second proxy, *Ap*, reflects the level of precipitation of particles (electrons and protons of the Solar wind, mainly) from the magnetosphere down to the lower thermosphere. These energetic particles also heat up the atmosphere, thereby affecting the local density. The particle number density depends on the activity level of the Sun itself (through coronal mass ejections), and the local variations in Earth's magnetic field. A typical quiescent value of *Ap* hovers around 0, but may rise to 400 and above (no units). Figures 1 and 2 show both the historic cyclic pattern in the *F10.7* value, and the characteristic *Ap* behavior during a single Solar cycle of 11 years.

The set of relevant equations are:

$$T = 900 + 2.5\,(F10.7 - 70) + 1.5\,Ap \quad \text{[Kelvin]}$$
$$m = 27 - 0.012\,(h - 200)$$
$$H = T / m \quad \text{[km]}$$
$$\rho = 6\times10^{-10} * \exp(-(h-175)/H) \quad [\text{kg m}^{-3}]$$

All constants were empirically derived to give an appropriate fit to the standard models (see [1]). Only the density has any physical relevance, the other quantities do not correspond to true atmospheric values at any height.

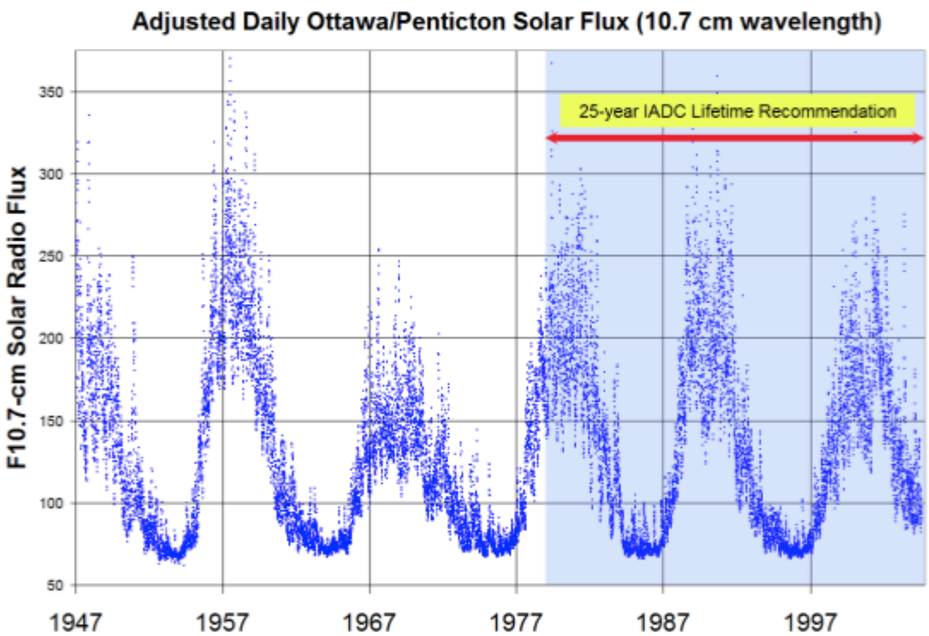

Figure 1. Historic *F10.7* values. Taken from [2].

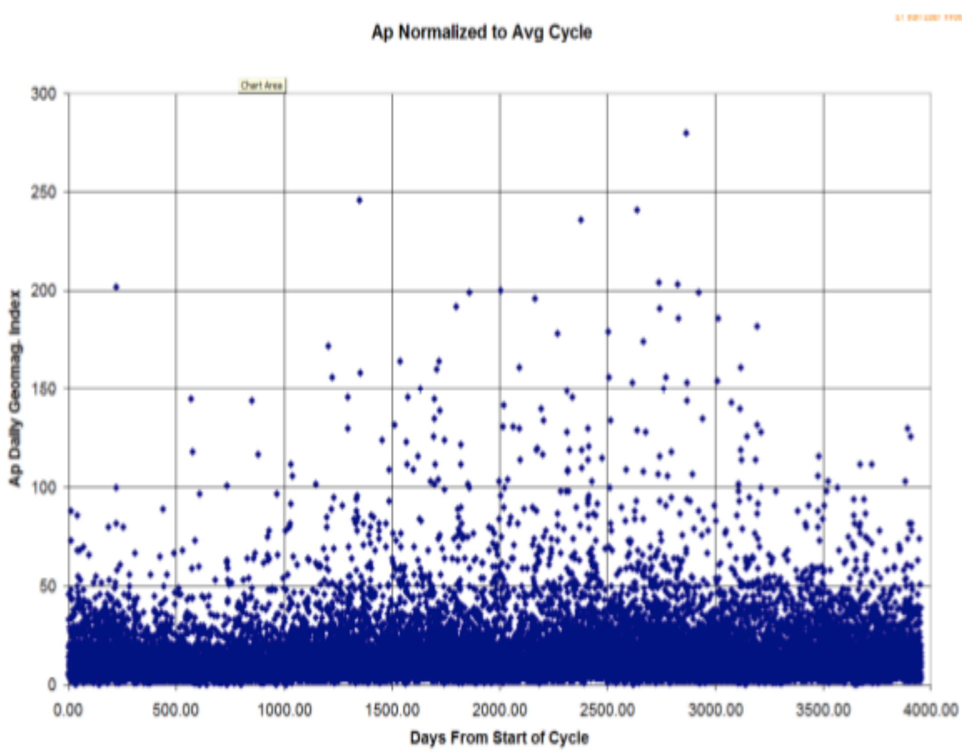

Figure 2. Historic *Ap* values. Taken from [2]

Satellite drag

The drag force is given by $D = ½ \rho v^2 A\ Cd$, where ρ is the local atmospheric density, v is the speed of the satellite, A is the cross-section of the object perpendicular to the motion, and Cd is a coefficient of drag. This last coefficient is generally assumed to be equal to two, but can vary wildly. Furthermore, A and Cd are commonly combined to form an effective cross-section Ae. For a circular orbit, we have the following relation between period and radius a: $P^2\ G\ M_{earth} = 4\ \pi^2\ a^3$. The reduction in period due to atmospheric drag is given by: $dP/dt = -3\pi a \rho\ (Ae\ /\ m)$. Combining this derivative with the other listed equations provides a framework in which one can estimate the orbital lifetime in an iterative fashion. Re-entry is assumed once the object's altitude has dropped below 180 km. At these altitudes all but the most massive of objects de-orbit within a few hours, thereby serving as a convenient end-point to the calculation.

Relative Calibration

Our altitude of interest is 700 km, more than the 500 km ceiling quoted in [1]. We therefore adjusted both the *F10.7* and *Ap* boundary conditions such that the results of the calculations match the three limiting curves in Figure B-1 from [2] (reproduced below) at an altitude of 650 km. We find that the combinations of *F10.7* and *Ap* listed in Table 1 produce comparable answers for an identical $CdA/m = 200$ drag case. Since we are looking at long decay time-scales at 700 km altitude, these values more or less represent the long-term average derived from Fig. 2. Our calculated estimates are over-plotted in Fig. 3.

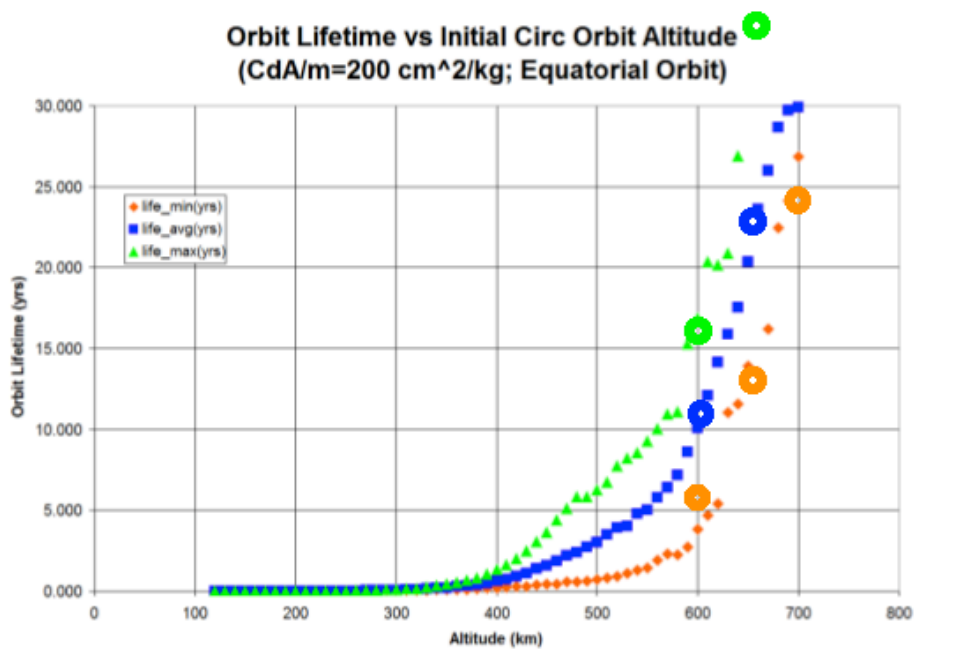

Figure 3. Orbital life-time estimates. Taken from [2]. Our calculations are plotted as open circles, using the same color-coding.

Table 1. Calibration results.

Calibration calculations; Surface Area=0.04 m^2, $CdA/m=200$ cm^2/kg						
Average [years]		Minimum [years]		Maximum [years]		
F10.7=160	*Ap*=10	*F10.7*=200	*Ap*=10	*F10.7*=135	*Ap*=10	Alt
11.2		6.4		16.8		600 km
23.4		12.9		36.9		650 km
46.9		24.6		63.1		700 km

Cubesat orbital lifetime estimates for nominal 700 km orbit

Based on these values, we arrive at the estimates for the orbital life-times listed in Table 2. We assume a 3U total mass of 4 kg, and a circular equatorial orbit of 700 km. Differences between equatorial and polar orbital lifetimes are probably not more than 10% at these altitudes (see Fig. 4). The surface area in column 4 is the actual maximum surface area that can be presented by the object. This number is then multiplied by the *Cd* value (assumed to be 2). Any orientation changes relative to the velocity vector will act to reduce the overall effective area. So, even though the baseline Cubesat with its 6x0.03 m^2 solar panels extended has a maximum area of 0.21 m^2, one should probably also consider the life-time estimates for 0.15 and 0.10 m^2. Note that for a circular sun-synchronous orbit, with the panels always pointing at the sun, the effective time-averaged cross-section for a baseline Cubesat is 0.14 m^2.

Table 2. Results for 700 km equatorial orbit

Orbital Life-time estimate			
Average [years]	Minimum [years]	Maximum [years]	Surface Area [m2]
57.0	31.1	>100	0.03 (3U only)
38.8	19.7	55.8	0.05
20.0	10.4	32.2	0.10
14.6	7.5	23.3	0.14 (Baseline eff)
13.6	7.0	22.0	0.15
9.9	5.0	16.0	0.21 (Baseline max)

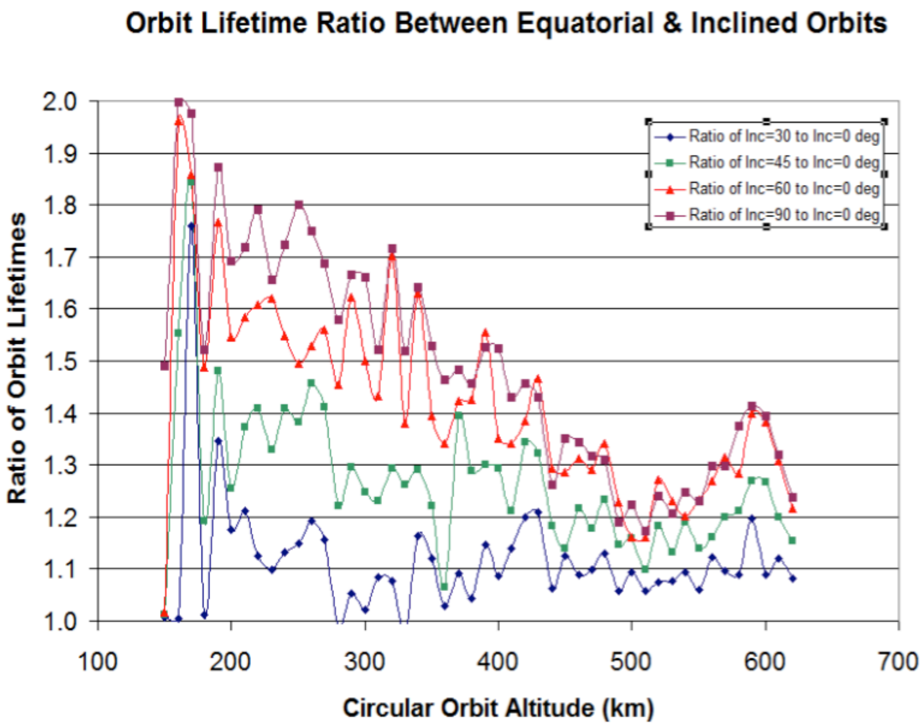

Figure 4. Ratios of orbital lifetimes as function of inclination. Taken from [2].

CubeSat orbital lifetime estimates for other regimes

The following section describes estimates for the orbital lifetime of a CubeSat across a variety of inclinations and altitudes. As before, the orbits are assumed to be circular. While at altitudes of 700 km and above the inclination dependence is small enough to be ignored, we do have to account for this at lower altitudes. Based on Fig. 4, we adopt the constants for the 45 and 90 degree inclination ratios given in Table 4. We have ignored the fine structure detail in the curves from Fig. 4, and used a linear fit instead.

Table 4. Orbital Lifetime ratios relative to Equatorial orbit.

Altitude [km]	45 degree inclination	90 degree inclination
600	1.17	1.25
550	1.19	1.31
500	1.22	1.38
450	1.26	1.43
400	1.29	1.49
350	1.32	1.56
300	1.35	1.62

Another change we have to make relative to the 700 km altitude case is that since the time-scales are short (a few years or less), we have to consider a larger *F10.7* variation (i.e., a running average of *F10.7* over 1 year shows larger variation than the numbers we were using for the 700 km case, see Table 1). We therefore bracket our runs with *F10.7*=70 and *F10.7*=200 values for the maximum and minimum cases respectively. The *Ap* index has been kept constant at 10. The results are listed in Table 5.

As a sanity check, we are considering Genesat 1, a 3U cubesat launched Dec 16, 2006 into a 415 km altitude orbit (inclination=40 degrees). It has a mass of 4.6 kg, and we furhter assume an effective surface area of 0.015 m^2 (half the 3U surface area). At the writing of this document (7/14/2010), it is still in orbit, albeit at a much reduced altitude of 270 km. Since it was launched near a solar minimum, we set the *F10.7* value to 80. The drag calculation then yields a 3.16 year decay time if it were in an equatorial orbit. Correcting for the inclination by a factor of ~1.28 (see Table 4) yields 4.04 years. This number cannot be too far off given that it is already down to 270 km, and is not expected to last much longer. In fact, JSPOC / SpaceTrack.org are predicting a decay date of 7/31/2010.

Table 5. Decay time-scales for a variety of altitudes and inclinations.

Altitude [km]	Effective Surface Area [m^2]	Equatorial Min [yr]	Max [yr]	i=45° Min [yr]	i=45° Max [yr]	i=90° Min [yr]	i=90° Max [yr]
550	0.02	5.96	40.32	7.09	47.98	7.81	52.82
550	0.03	4.01	30.65	4.77	36.47	5.25	40.15
550	0.05	2.43	18.47	2.89	21.98	3.18	24.20
550	0.10	1.22	9.95	1.45	11.84	1.60	13.03
550	0.15	0.82	6.74	0.98	8.02	1.07	8.83
550	0.21	0.58	4.88	0.69	5.81	0.76	6.39
500	0.02	2.68	15.98	3.27	19.50	3.70	22.05
500	0.03	1.79	10.94	2.18	13.35	2.47	15.10
500	0.05	1.08	6.80	1.32	8.30	1.49	9.38
500	0.10	0.54	3.49	0.66	4.26	0.75	4.82
500	0.15	0.36	2.35	0.44	2.87	0.50	3.24
500	0.21	0.26	1.68	0.32	2.05	0.36	2.32
450	0.02	1.14	5.43	1.44	6.84	1.63	7.76
450	0.03	0.76	3.67	0.96	4.62	1.09	5.25
450	0.05	0.46	2.23	0.58	2.81	0.66	3.19
450	0.10	0.23	1.12	0.29	1.41	0.33	1.60
450	0.15	0.15	0.75	0.19	0.94	0.21	1.07
450	0.21	0.11	0.54	0.14	0.68	0.16	0.77
400	0.02	0.46	1.66	0.59	2.14	0.69	2.47
400	0.03	0.31	1.11	0.40	1.43	0.46	1.65
400	0.05	0.18	0.70	0.23	0.90	0.27	1.04
400	0.10	0.09	0.34	0.12	0.44	0.13	0.51
400	0.15	0.06	0.22	0.08	0.28	0.09	0.33
400	0.21	0.04	0.16	0.05	0.21	0.06	0.24

350	0.02	0.17	0.46	0.22	0.61	0.27	0.72
350	0.03	0.12	0.31	0.16	0.41	0.19	0.48
350	0.05	0.07	0.19	0.09	0.25	0.11	0.30
350	0.10	0.04	0.09	0.05	0.12	0.06	0.14
350	0.15	0.02	0.06	0.03	0.08	0.03	0.09
350	0.21	0.02	0.05	0.03	0.07	0.03	0.08
300	0.02	0.06	0.12	0.08	0.16	0.10	0.19
300	0.03	0.04	0.08	0.05	0.11	0.06	0.13
300	0.05	0.02	0.05	0.03	0.07	0.03	0.08
300	0.10	0.01	0.02	0.01	0.03	0.02	0.03
300	0.15	0.01	0.02	0.01	0.03	0.02	0.03
300	0.21	0.01	0.01	0.01	0.01	0.02	0.02

It should be noted that the Min and Max values only have meaning if the resulting time-scale is small compared to the 11 year solar cycle. If this is the case, then the Min and Max values represent the extremes one can encounter during the Solar activity cycle (see Fig. 2). Where the time-scales are larger than a few years, and especially larger than the 11 years Solar cycle, the initial assumption of average *F10.7* values of 200 and 70, for Min and Max respectively, are wrong. The reader is referred to Table 6, in which we have listed the expected *average* orbital life-time for these cases using a mean *F10.7* of 160 (cf. Table 1).

Table 6. Decay time-scales for a variety of altitudes and inclinations – long time scale estimates.

Altitude [km]	Effective Surface Area [m²]	Equatorial Average [yr]	i=45° Average [yr]	i=90° Average [yr]
550	0.02	9.83	11.70	12.88
550	0.03	6.67	7.94	8.74
550	0.05	4.06	4.83	5.60
500	0.02	4.19	5.11	5.78
500	0.03	2.82	3.44	3.89
500	0.05	1.70	2.07	2.35

References

[1] "Satellite Orbital Decay Calculations", 1999, Australian Space Weather Agency, http://www.ips.gov.au
[2] "Space Systems – determining orbit lifetime", 2007, ISO International Standard working document, ref ISO/WD/27852

Auspices

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